

NUMERICAL ALGEBRA,
CONTROL AND OPTIMIZATION

doi:10.3934/naco.2019043

**RESOURCE ALLOCATION AND TARGET SETTING
BASED ON VIRTUAL PROFIT IMPROVEMENT**

JAFAR SADEGHI*

Department of Mathematics
Faculty of Mathematical Sciences and Computer
Kharazmi University, Tehran, Iran

MOJTABA GHIYASI

Faculty of Industrial Engineering and Management Sciences
Shahrood University of Technology, Shahrood, Iran

AKRAM DEHNOKHALAJI

, Operations and Information Management Department
Aston Business School, Aston University, Birmingham, B4 7ET, UK

(Communicated by Gerhard-Wilhelm Weber)

ABSTRACT. One application of Data Envelopment Analysis (DEA) is the resource allocation and target setting among homogeneous Decision Making Units (DMUs). In this paper, we assume that all units are under the supervision and control of a central decision making unit, for instance chain stores, banks, schools, etc. The aim is to allocate available resources among units in a way that the so-called organisational overall "*virtual profit*" is maximized. Our method is highly flexible in decision making to achieve the goals of the Decision Maker (DM). The resulting production plans maintain the following characteristics: (1) the virtual profit of each unit is calculated with a common set of weights; (2) the selected weights for calculating the virtual profit prevent the virtual profit of the system from getting worse; (3) the virtual profits of less profitable units are improved as much as possible. The proposed method is illustrated with a simple numerical example and a real life application.

1. Introduction. Mathematical Optimization is a branch of applied mathematics which is useful in many different fields. For examples: Manufacturing, production, inventory control, transportation, economics, marketing and etc., (For some of the recent applications of optimisation, see [22, 23, 24, 26, 25]).

One field in optimization is Data Envelopment Analysis (DEA). DEA is an effective approach to evaluate the performance of homogeneous Decision Making Units (DMUs), first proposed by Charnes et al. [4]. In recent years, several studies have been developed regarding the applications of DEA in educational institutes, industries, banks, etc. (For some of the recent applications using DEA, see

2010 *Mathematics Subject Classification.* Primary: 97M40, 90B50; Secondary: 90C90, 90C05.

Key words and phrases. Data envelopment analysis, resource allocation, target setting; common weights, profit.

We thank four reviewers of this journal for their most constructive comments.

* Corresponding author: E-mail addresses: std.j.sadeghi@khu.ac.ir.

[10, 29, 13, 13, 34]). One important application of DEA is the resource allocation and target setting among DMUs where the DM allocates available resources to units to achieve a specific goal. The goal is defined differently in various resource allocation studies.

Golany et al. [11] suggested an approach by emphasizing on the importance of resource allocation as a tool to improve the effectiveness of units utilizing goal programming techniques. Golany and Tamir [12] presented a linear programming model for resource allocation with the aim of maximizing total outputs. Their model can be used for single output cases. For multiple output cases, they utilized subjective weights for outputs. They proposed to solve the obtained linear programming model by Dantzing-Wolf decomposition algorithm. Athanassopoulos [1] used Goal programming and DEA (GODEA) for resource allocation and target setting in a multi-level programming problem. Yan et al. [33] extended the preference cone constraints in an inverse DEA model proposed by Wei et al. [30]. Using Multi-Objective Linear Programming (MOLP) and weighted sum single-objective programming problems, when some or all inputs and outputs are changed, they estimated inputs and outputs which do not change the efficiency scores (without making any changes on efficiencies). Beasley [2] proposed a non-linear model by maximizing the efficiency average of units for resource allocation and output target setting in which inputs and outputs are characterized for the next period, simultaneously. Considering all units under the supervision of a central unit, Lozano and Villa [20] proposed a centralized model which projects all units onto the efficient frontier simultaneously through reducing the total input consumption without reducing the total outputs. In this regard, they presented a centralized target setting model to assign available containers to a number of municipalities. Korhonen and Syrjanen [15] and Dehnokhalaji et al. [5] presented resource allocation approaches based on DEA and MOLP techniques.

Xiaoya and Jinchuan [32] suggested a framework for resource allocation combining the concept of return to scale, inverse DEA and common weights. Wu et al. [31] presented an approach for resource allocation considering environmental factors i.e., undesirable inputs and outputs. Their main goal is to maximize desirable outputs and to minimize undesirable outputs. Also, the reader can see more resources allocation papers in [6, 21, 18, 8, 14, 5, 16, 17].

However, most researchers in DEA literature have addressed centralized resource allocation models from the reallocation perspective i.e., allocating the current resources to units as well. However, in some organizations, there is a situation in which the amount of the current resources should be increased (i.e., allocating some additional amount of resources among units) or reduced (i.e., allocating less amount of resources among units), in the next production period.

It seems that, to date, the extra or shortage resource-allocation problem has not been addressed sufficiently in literature. For example, Beasley [2], Korhonen and Syrjanen [15], Nasrabadi et al. [21] and Sadeghi and Dehnokhalaji [27] have addressed these issues. This kind of extra or shortage resource-allocation problem can be frequently observed in practice. For example, the top manager of bank branches aims to assign a large amount of premium to their branches; a company aims to allocate some bonus to selected staff members at the end of the year; a factory needs to reduce labor force because of being on a tight budget. This study aims to answer this question: How should we distribute the premium, bonus or

allocate less labor among units to achieve the fair principle and meanwhile make the all beneficial?

In this paper, we consider a centralized resource allocation and target setting approach. First, we introduce the concept of Common Weight profit (CW-profit) and provide its interpretation and then we define the overall CW-profit of the organization applying the SBM model. We prove that the sum of CW-profits of units is equal to the overall CW-profit of the organization where input-output weights are common among all units. One advantage of non-radial SBM model to traditional radial CCR and BCC models is that the slacks and surpluses of inputs and outputs are also considered in calculating the efficiency scores. In other words, the mixed inefficiency can be distinguished from the technical efficiency in SBM model. Moreover, positive input-output weights are obtained by solving this model that are independent of the chosen values for ϵ in CCR and BCC models.

Our proposed method has high flexibility in applying the DM's decisions and viewpoints. Regarding the DM's aims, our method generates production plans that maximize the overall CW-profit of the organization by assuming a common set of weights. Since the overall CW-profit of all units can be affected by the units with the worst behaviour, our method improves CW-profits of units as much as possible and meanwhile prevents the CW-profit of the each unit from becoming worse. Finally, different cases of allocation including allocation of extra resources, resource reallocation and allocation of less resources are investigated by a simple numerical example and then by a real-life application.

The rest of this paper is organized as follows. Section 2 presents an approach to calculate the CW-profit of DMUs. In section 3, we first formulate a model to calculate the maximum profit of the central unit and then another model is presented for resource allocation and target setting plans of the next period. This section ends by a numerical example. An application is presented in section 4 and section 5 concludes the results.

2. Technology: basic preliminaries. Consider n homogeneous DMU_j , $j = 1, \dots, n$ with input values x_{ij} and outputs y_{rj} where $i = 1, \dots, m$, $r = 1, \dots, s$. These units are under the supervision of a central decision maker. This situation happens when all units belong to the same organization. Many traditional applications of DEA (bank branches, hospitals, university departments, secondary schools, police stations, etc.) belong to this category (Lozano et al. [19]).

The famous Production Possibility Set (PPS) in DEA is denoted as:

$$T = \left\{ (x, y) \mid \sum_{j=1}^n \lambda_j x_j \leq x, \sum_{j=1}^n \lambda_j y_j \geq y, \lambda \in \Gamma \right\},$$

where $\Gamma = R_n^+$ denotes the PPS with the constant returns to scale (CRS) assumption, and $\Gamma = \{\lambda \in R_n^+ \mid \sum_{j=1}^n \lambda_j = 1\}$ denotes the PPS with the variable returns to scale (VRS) assumption.

One basic non-radial models to evaluate the efficiency score of units in DEA is SBM model proposed by Tone [28]. In case of non-discretionary inputs, they are assumed to be exogenously fixed and discretionary inputs can be adjusted proportionally. Therefore, the SBM model for evaluating the efficiency of DMU_p is written as follows:

$$\begin{aligned}
\min \rho &= \frac{1 - \frac{1}{m} \sum_{i \in D} \frac{s_i^-}{x_{ip}}}{1 + \frac{1}{s} \sum_{r=1}^s \frac{s_r^+}{y_{rp}}} \\
s.t. \quad &\sum_{j=1}^n \lambda_j x_{ij} + s_i^- = x_{ip}, \quad i \in D \cup ND, \\
&\sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{rp}, \quad r = 1, \dots, s, \\
&\lambda_j \geq 0, \quad j = 1, \dots, n, \\
&s_i^- \geq 0, s_r^+ \geq 0, \quad i = 1, \dots, m, \quad r = 1, \dots, s,
\end{aligned} \tag{1}$$

where D and ND stands for the discretionary and non-discretionary input indices, respectively. As it can be seen in the objective function of model (1), non-discretionary inputs have no roles in assessing the efficiency score of DMU_p .

Suppose that ρ^* is the optimal value of objective function of model (1). Then $0 \leq \rho^* \leq 1$. Also DMU_p is efficient if and only if $\rho^* = 1$ (See Tone [28] for more details).

3. An approach to calculate the CW-profit of a unit. Although the DM is interested in increasing the overall profit of the organization through resource allocation plans, he (she) is also concerned with the overall consumption of different inputs, i.e, $\sum_{j=1}^n x_{ij}$, for all i and the overall production of different outputs, i.e, $\sum_{j=1}^n y_{rj}$, for all r . Hence the DM considers a virtual unit with i -th input equal to summation of i -th inputs of all units, $x_i^c = \sum_{j=1}^n x_{ij}$ and r -th output equal to summation of r -th outputs of all units, $y_r^c = \sum_{j=1}^n y_{rj}$. For simplicity, the activity vector of central unit is represented by (X^c, Y^c) .

Applying Charnes-Cooper transformation (Charnes and Cooper [3]) on model (1) and writing the dual from of the obtained linear programming model we have the following model:

$$\begin{aligned}
&\max \xi \\
s.t. \quad &\xi + \sum_{i=1}^m v_i x_{ip} - \sum_{r=1}^s u_r y_{rp} = 1, \\
&\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, & j = 1, \dots, n, \\
&u_r \geq \frac{\xi}{s y_{rp}}, & r = 1, \dots, s, \\
&v_i \geq \frac{1}{m x_{ip}}, & i \in D, \\
&v_i \geq 0, & i \in ND.
\end{aligned} \tag{2}$$

Below, the equivalent problem is obtained by eliminating ξ from model (2):

$$\begin{aligned}
\max \quad & \sum_{r=1}^s u_r y_{rp} - \sum_{i=1}^m v_i x_{ip}, \\
s.t. \quad & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \\
& u_r \geq \frac{1 + \sum_{r=1}^s u_r y_{rp} - \sum_{i=1}^m v_i x_{ip}}{s y_{rp}}, \quad r = 1, \dots, s, \\
& v_i \geq \frac{1}{m x_{ip}}, \quad i \in D, \\
& v_i \geq 0, \quad i \in ND.
\end{aligned} \tag{3}$$

In the dual form of SBM model, v_i 's and u_r 's, that are always positive, can be interpreted as the "virtual" input costs and "virtual" output prices, respectively (Tone [28]).

The aim of the dual problem is to obtain the optimal virtual prices for the DMU_p so that the virtual profit of all units (including DMU_p) do not exceed zero and the "virtual" profit of DMU_p is maximized. Note that, in the objective function of model (1), the total cost of inputs have been subtracted from the total revenue of outputs for DMU_p , which provides its net profit. Also according to the first constraint of model (3), the highest profit is equal to zero for each unit.

Suppose that v_i^* and u_r^* are the optimal i -th input cost and r -th output revenue in evaluating central unit (X^c, Y^c) by model (3). We have the following definitions:

Definition 3.1. The Common Weight profit (CW-profit) of the central unit is defined by $\sum_{r=1}^s u_r^* y_r^c - \sum_{i=1}^m v_i^* x_i^c$.

Definition 3.2. The value $\sum_{r=1}^s u_r^* y_{rj} - \sum_{i=1}^m v_i^* x_{ij}$ is called the CW-profit of DMU_j .

The objective function of model (3) can be rewritten as follows:

$$\begin{aligned}
\sum_{r=1}^s u_r^* y_r^c - \sum_{i=1}^m v_i^* x_i^c &= \sum_{r=1}^s u_r^* \sum_{j=1}^n y_{rj} - \sum_{i=1}^m v_i^* \sum_{j=1}^n x_{ij} \\
&= \sum_{j=1}^n \left(\sum_{r=1}^s u_r^* y_{rj} - \sum_{i=1}^m v_i^* x_{ij} \right).
\end{aligned}$$

The above equation shows that the CW-profit of the central unit is equal to the summation of CW-profits of all units, as expected. Also, the central unit has the highest profit, namely zero, if and only if the virtual profits of all units are equal to zero. Applying model (3) for evaluating the central unit, the overall CW-profit is maximized choosing the same weights for inputs and outputs for all units. However, since all units are assumed to be homogenous and under the supervision of the same organization, there is no rationale for using endogenous individual weights in calculation of profits.

The DM can identify the units with less profit based on the results of model (3) and can increase the overall CW-profit by allocating resources to them.

4. The proposed method for resource allocation and target setting. In this section, a novel method is developed to generate production plans by resource allocation and target setting among DMUs. To this end, we first formulate a model to compute the maximum CW-profit of the central unit where inputs and outputs can vary in the range suggested by the DM. Then, we build up another model that identifies the production plans for the next period in a way that the CW-profit of the central unit remains maximum and CW-profits of units with the lowest profit are improved as much as possible.

4.1. Resource allocation to maximize the CW-profit of the central unit.

Suppose that $(\nabla x_1^c, \dots, \nabla x_m^c)$ is the vector of input changes for the central unit. Therefore, $(\bar{x}_1^c, \dots, \bar{x}_m^c) = (x_1^c + \nabla x_1^c, \dots, x_m^c + \nabla x_m^c)$ are the planed resources for the next period. Also, let \bar{x}_{ij} denotes i -th input of j -th unit after resource allocation. Set,

$$\begin{aligned}\alpha_i &= \frac{\bar{x}_i^c}{x_i^c}, & i \in D, \\ \alpha_i &= 1, & i \in ND, \\ \delta_{ij} &= \frac{\bar{x}_{ij}}{x_i^c}, & i \in D, j = 1, \dots, n,\end{aligned}\tag{4}$$

where α_i denotes the proportion of i -th current resource which should be allocated. If the input value in the next period is smaller than, greater than or equal to the current input value, then α_i is smaller than, greater than or equal to one, respectively. Also, decision variable $\delta_{ij} \geq 0$ denotes the fraction of i -th current resource that should be allocated to DMU_j as its i -th input.

The equation $\bar{x}_i^c = \sum_{j=1}^n \bar{x}_{ij}$ should be hold in order to achieve the goals of the DM in allocation. Therefore, the following equation is obtained from (4):

$$\alpha_i = \sum_{j=1}^n \delta_{ij}, \quad i \in D.\tag{5}$$

Similarly, suppose that $(\bar{y}_1^c, \dots, \bar{y}_s^c)$ and \bar{y}_{rj} denote the output value of the central unit and r -th new output of DMU_j after allocation, respectively. We assume that

$$\begin{aligned}\beta_r &\leq \frac{\bar{y}_r^c}{y_r^c} \leq \gamma_r, & r = 1, \dots, s, \\ \pi_{rj} &= \frac{\bar{y}_{rj}}{y_r^c}, & j = 1, \dots, n, r = 1, \dots, s,\end{aligned}\tag{6}$$

where β_r and γ_r show the lower and upper bounds for the proportional changes of r -th current output that is determined by the DM for target setting, respectively. Also, decision variable $\pi_{rj} \geq 0$ shows the fraction of y_r^c that should be set as a goal for the r -th output of DMU_j . The equation $\bar{y}_r^c = \sum_{j=1}^n \bar{y}_{rj}$ holds in order to achieve the goals of the DM in allocation. Hence, the following equation is obtained from (6):

$$\bar{y}_r^c = \sum_{j=1}^n \pi_{rj} y_r^c, \quad r = 1, \dots, s.\tag{7}$$

The maximum CW-profit of the central unit before allocation can be obtained by solving model (3) assessing the central unit (X^c, Y^c) . To obtain the maximum CW-profit of the central unit in resource allocation plans, we formulate model (8) based on model (3) and considering inputs and outputs changes according to (4), (5), (6)

and (7). The general model for input allocation and output target setting along with maximizing CW-profit of the central unit is presented as follows:

$$\begin{aligned}
\max \quad & z = \sum_{j=1}^n \left(\sum_{r=1}^s u_r \pi_{rj} y_r^c - \sum_{i \in D} v_i \delta_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \right) \\
s.t \quad & \sum_{r=1}^s u_r \pi_{rj} y_r^c - \sum_{i \in D} v_i \delta_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \\
& u_r \geq \frac{1+z}{s \sum_{j=1}^n \pi_{rj} y_r^c}, \quad r = 1, \dots, s, \\
& v_i \geq \frac{1}{m \sum_{j=1}^n \delta_{ij} x_i^c}, \quad i \in D, \\
& \sum_{j=1}^n \delta_{ij} = \alpha_i, \quad i \in D, \\
& \gamma_r \leq \sum_{j=1}^n \pi_{rj} \leq \beta_r, \quad r = 1, \dots, s, \\
& L_{ij}^I \leq \frac{\delta_{ij} x_i^c}{x_{ij}} \leq U_{ij}^I, \quad i \in D, j = 1, \dots, n, \\
& L_{rj}^o \leq \frac{\pi_{rj} y_r^c}{y_{rj}} \leq U_{rj}^o, \quad r = 1, \dots, s, j = 1, \dots, n, \\
& q_{ij} \geq 0, p_{rj} \geq 0, \\
& v_i \geq 0, \quad i \in ND,
\end{aligned} \tag{8}$$

where, α_i 's, γ_r 's, β_r 's, U_{rj}^o 's, L_{rj}^o 's, U_{ij}^I 's and L_{ij}^I 's are parameters given by the DM. Note that, model (8) deals with extra resource allocation problem if $\alpha_i > 1$. If $\alpha_i < 1$, the corresponding inputs are decreased and if $\alpha_i = 1$, we deal with the resource reallocation problem. L_{ij}^I and U_{ij}^I denote the lower bounds and upper bounds of possible changes in i -th resource of DMU_j , respectively. In general, the DM does not assume predetermined values for outputs, but he (or she) can determine target setting for units according to his (her) knowledge. Therefore, β_r and γ_r values denote upper and lower bounds for r -th total output of the central unit, respectively. This means that a range is provided for each total output after allocation by the DM. Also L_{rj}^o and U_{rj}^o denote upper and lower bounds, characterized by the DM for target setting for r -th output of DMU_j . For instance, if the DM is interested in keeping output values unchanged, $\beta_r = \gamma_r = 1$ and $L_{rj}^o = U_{rj}^o = 1$ for all r and j .

Model (8) is a nonlinear but it can be written as a linear model by defining $q_{ij} = v_i \delta_{ij}$ and $p_{rj} = u_r \pi_{rj}$, as follows:

$$\begin{aligned}
\max \quad & z = \sum_{j=1}^n \left(\sum_{r=1}^s p_{rj} y_r^c - \sum_{i \in D} q_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \right) \\
s.t \quad & \sum_{r=1}^s p_{rj} y_r^c - \sum_{i \in D} q_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \\
& \sum_{j=1}^n p_{rj} \geq \frac{1+z}{s y_r^c}, \quad r = 1, \dots, s,
\end{aligned}$$

$$\begin{aligned}
\sum_{j=1}^n q_{ij} &\geq \frac{1}{mx_i^c}, \quad i \in D, \\
\sum_{j=1}^n q_{ij} &= \alpha_i v_i, \quad i \in D, \\
\gamma_r u_r &\leq \sum_{j=1}^n p_{rj} \leq \beta_r u_r, \quad r = 1, \dots, s, \\
L_{ij}^I v_i x_{ij} &\leq q_{ij} x_i^c \leq U_{ij}^I v_i x_{ij}, \quad i \in D, j = 1, \dots, n, \\
L_{rj}^O u_r y_{rj} &\leq p_{rj} y_r^c \leq U_{rj}^O u_r y_{rj}, \quad r = 1, \dots, s, j = 1, \dots, n, \\
q_{ij} &\geq 0, p_{rj} \geq 0, \\
v_i &\geq 0, \quad i \in ND.
\end{aligned} \tag{9}$$

By solving model (9), δ_{ij} and π_{rj} values for i -th input and r -th output of DMU_j can be obtained through $\delta_{ij} = \frac{q_{ij}}{v_i}$ and $\pi_{rj} = \frac{p_{rj}}{u_r}$. Hence, the maximum CW-profit of the central unit can be obtained in a way that resources are allocated to DMUs and the DM's target settings for outputs are achieved.

4.2. Resource allocation to improve the CW-profit of units with less virtual profits. After introducing model (9) to calculate the maximum CW-profit, we are able to, formulate a model to generate plans for resource allocation and target setting. Before presenting the model, basic criteria for resource allocation and target setting are reported as follows:

- (1) The result of resource allocation and target setting plans for each unit should be evaluated with a common set of weights. Since the units are assumed to be homogeneous and working in a same organization, under the supervisor of the central unit, it is not rationale to choose the weights separately.
- (2) The resulting plan, namely the chosen weights for evaluating DMUs, should not make the CW- profit of the central unit worse. This requirement guarantees the stability of the decision and ensures the DM that his (her) production plans do not worsen the overall CW-profit. The DM is interested to allocate resources to units in away that the CW-profit of the central unit, in the best case, is not getting worse.
- (3) The CW-profit of the units with the worst behaviour in the organization should be improved as much as possible. In this way, the gap between the profit of units can be limited. Moreover, if the performance of the central unit is highly related to the units that have the worst behaviour, the total performance within the organization is improved as a consequence.

With the above mentioned criteria, the proposed model is presented as follows:

$$\begin{aligned}
\max \quad & \min_j \left\{ w_j \left(\sum_{r=1}^s p_{rj} y_r^c - \sum_{i \in D} q_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \right) \right\} \\
s.t \quad & z = \sum_{j=1}^n \left(\sum_{r=1}^s p_{rj} y_r^c - \sum_{i \in D} q_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \right) \geq z^*, \\
& \sum_{r=1}^s p_{rj} y_r^c - \sum_{i \in D} q_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \leq 0, \quad j = 1, \dots, n,
\end{aligned}$$

$$\begin{aligned}
\sum_{j=1}^n p_{rj} &\geq \frac{1+z}{sy_r^c}, \quad r = 1, \dots, n, \\
\sum_{j=1}^n q_{ij} &\geq \frac{1}{mx_i^c}, \quad i \in D, \\
\sum_{j=1}^n q_{ij} &= \alpha_i v_i, \quad i \in D, \\
\gamma_r u_r &\leq \sum_{j=1}^n p_{rj} \leq \beta_r u_r, \quad r = 1, \dots, s, \\
L_{ij}^I v_i x_{ij} &\leq q_{ij} x_i^c \leq U_{ij}^I v_i x_{ij}, \quad i \in D, j = 1, \dots, n, \\
L_{rj}^O u_r y_{rj} &\leq p_{rj} y_r^c \leq U_{rj}^O u_r y_{rj}, \quad r = 1, \dots, s, j = 1, \dots, n, \\
q_{ij} &\geq 0, p_{rj} \geq 0, \\
v_i &\geq 0, \quad i \in ND,
\end{aligned} \tag{10}$$

where z^* is the maximum value of the CW-profit of the central unit, obtained by solving model (9). Also the vector (w_1, \dots, w_n) with non-negative entries satisfying $\sum_{k=1}^n w_k = 1$, is the preference or weight vector of the DM for the desirability of the CW-profit of each unit.

The objective function of model (10) maximizes the minimal weighted CW-profit of units. The constraints include the decision variables for generating resource allocation and target setting plans. Note that, the first constraint of model (10) ensures that the overall CW-profit is not getting worse. In addition, the last two set of constraints of model (10) remain to limit the resources consumption. The solution of model (10) is not unique. Such solutions are called efficient solutions in Multiple-Criteria Decision Making (MCDM) literature. In such circumstances, each efficient solution is acceptable ([15]). We are able to transform it to a single-objective linear model by introducing the parameter η as follows:

$$\begin{aligned}
&\max \quad \eta \\
&s.t. \quad \eta \leq w_j \left(\sum_{r=1}^s p_{rj} y_r^c - \sum_{i \in D} q_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \right), \quad j = 1, \dots, n, \\
&\quad z = \sum_{j=1}^n \left(\sum_{r=1}^s p_{rj} y_r^c - \sum_{i \in D} q_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \right) \geq z^*, \\
&\quad \sum_{r=1}^s p_{rj} y_r^c - \sum_{i \in D} q_{ij} x_i^c - \sum_{i \in ND} v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \\
&\quad \sum_{j=1}^n p_{rj} \geq \frac{1+z}{sy_r^c}, \quad r = 1, \dots, n, \\
&\quad \sum_{j=1}^n q_{ij} \geq \frac{1}{mx_i^c}, \quad i \in D, \\
&\quad \sum_{j=1}^n q_{ij} = \alpha_i v_i, \quad i \in D,
\end{aligned}$$

$$\begin{aligned}
\gamma_r u_r &\leq \sum_{j=1}^n p_{rj} \leq \beta_r u_r, \quad r = 1, \dots, s, \\
L_{ij}^I v_i x_{ij} &\leq q_{ij} x_i^c \leq U_{ij}^I v_i x_{ij}, \quad i \in D, \quad j = 1, \dots, n, \\
L_{rj}^O u_r y_{rj} &\leq p_{rj} y_r^c \leq U_{rj}^O u_r y_{rj}, \quad r = 1, \dots, s, \quad j = 1, \dots, n, \\
q_{ij} &\geq 0, p_{rj} \geq 0, \\
v_i &\geq 0, \quad i \in ND.
\end{aligned} \tag{11}$$

Now, we illustrate the outcome of model (11) by a simple numerical example.

4.3. Numerical Example. Consider 7 units with two inputs and one output. The data are shown in Table 1. Our resource allocation is done in two cases where we assume that the DM considers the same preference for all units, i.e. $w_1 = w_2 = \dots = w_7 = \frac{1}{7}$.

Case I: Let first input is the only discretionary input, i.e. $D = \{1\}$. In this case, the DM's policy is to decrease the amount of the first resource by 10% i.e., the first input of central unit should be decreased by 10%, so $\alpha_1 = 0.9$. Also, the amount of the first input changes for all units is 20% or equivalently, the first input of the units can be decreased or increased by 20%. Therefore $L_{1j}^I = 0.8$ and $U_{1j}^I = 1.2$. Also we assume that the output of the central unit can be reduced by 10% i.e, $\gamma_1 = 0.9$ and $\beta_1 = 1$. Furthermore, the amount of output changes for all units is 5%. Then we have, $U_{1j}^O = 1.05$ and $L_{1j}^O = 0.95$.

Case II: Let both inputs are under discretionary of the DM, i.e. $D = \{1, 2\}$. In this case, the DM's Policy is reallocation of resource for units. Hence, the amount of the resources do not change so $\alpha_i = 1$. Also, inputs values of the units can be changed by 20% i.e, $L_{ij}^I = 0.8$ and $U_{ij}^I = 1.2$. Also, we assume that the output of the central unit and units can be increased at most to size 10% and 5%, respectively. Then we have, $\gamma_1 = 1$, $\beta_1 = 1.10$ and $U_{1j}^O = 1.05$, $L_{1j}^O = 1$.

Input-output values for units after resource allocation case I and II are reported in Table 1 in both cases. The CW-profit values and efficiency scores of units are re-

TABLE 1. Data set and results of numerical example

DMU				Case I			Case II		
	I_1	I_2	O	I_1	I_2	O	I_1	I_2	O
A	8	24	4	6.4	24	4.1	6.4	21.32	4.2
B	27	27	9	32.4	27	8.55	32.4	32.4	9
C	56	8	8	54.3	8	7.6	67.2	9.6	8
D	8	14	2	6.4	14	2.1	6.4	11.2	2.1
E	42	24	6	33.6	24	6.3	33.6	20.28	6.3
F	32	8	4	25.6	8	4.2	30.18	9.6	4.2
G	30	3	3	24	3	3.15	26.82	3.6	3.15
Central	203	108	36	182.7	108	36	203	108	36.95

ported in Table 2. The values p_j and p'_j , obtained by solving model (3) in evaluating central unit, represent the CW-profit of the DMU_j before and after the allocation, respectively. Also, The values θ_j and θ'_j , obtained by solving model (2), represent the efficiency scores of the DMU_j before and after the allocation, respectively.

As it can be seen in Table 2, the efficiency score of all units and also the central unit have been increased after the allocation and efficient units remains efficient.

The CW-profit of each unit and also the organization increased in both cases. The Farrell's frontiers ([9]) shown in figure 1 and 2, before and after allocation, respectively. As it can be seen, units A , B and C are on the strong frontier after allocation. Inefficient units D , E and F are closer to the new frontier in both cases and unit G that was located on the weak efficient frontier before allocation, is on the strong efficient frontier after allocation. Furthermore, the central unit is closer to the frontier in both cases.

Note that the production plans after the allocation may create different production possibility sets. However, by regarding the ranges for the inputs and output changes, the feasibility of the production plans is guaranteed.

TABLE 2. Results of numerical example

DMU	Case I				Case II			
	p_j	p'_j	θ_j	θ'_j	p_j	p'_j	θ_j	θ'_j
A	-0.0457	-0.0239	1	1	-0.0457	-0.0091	1	1
B	0	0	1	1	0	0	1	1
C	-0.0047	0	1	1	-0.0047	0	1	1
D	-0.042	-0.0287	0.63	0.7	-0.042	-0.0151	0.59	0.73
E	-0.0869	-0.0459	0.59	0.77	-0.0869	-0.0151	0.59	0.91
F	-0.0307	-0.0037	0.75	0.97	-0.0307	-0.0099	0.69	0.9
G	-0.0239	-0.0028	0.85	1	-0.0239	0	0.85	1
Central	-0.234	-0.1049	0.77	0.9	-0.234	-0.0491	0.77	0.95

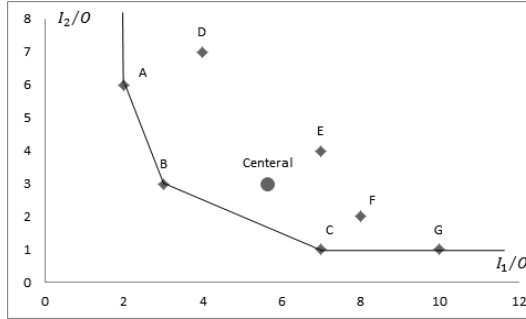


FIGURE 1. Farrell's frontier before resource allocation.

5. An application. In this section, we present how our proposed resource allocation model works and the resource to the units and its results are suitable in practice, using the data for 25 supermarkets in Finland taken from [15].

The supermarkets belong to the same chain and under the supervision of the central unit that has the power of the control their performance can allocate resources to them. The information about inputs (Man Hours and Size) and outputs (Sales and Profit) are provided in Table 3. We consider two different cases of resource allocation, in the first case, the DM does not have the discretion to change the size of the supermarkets i.e. $D = \{1\}$. The DM's policy is increasing the total Man Hours for all supermarkets by 10%, hence $\alpha_1 = 1.10$. To ensure the managerial feasibility, the change of unit inputs is limited to a 10% decreasing and a 10% increasing, so $L_{ij}^I = 0.9, U_{ij}^I = 1.1$. Also, we assume that total outputs have at most 10% ability

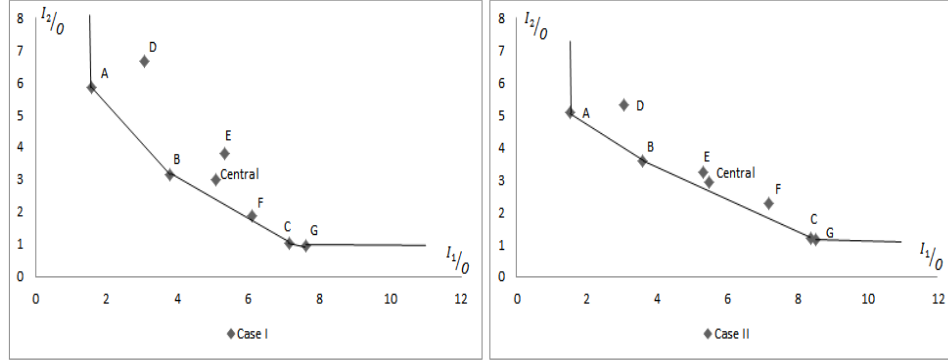


FIGURE 2. Farell's frontier after resource allocation in cases I and II.

TABLE 3. Data set and results of numerical example

DMU					Case I				Case II			
	I_1	I_2	O_1	O_2	I_1	I_2	O_1	O_2	I_1	I_2	O_1	O_2
1	79.1	4.99	115.3	1.71	87.01	4.99	121.06	1.8	71.19	4.49	121.06	1.8
2	60.1	3.3	75.2	1.81	66.11	3.3	78.96	1.9	54.09	2.97	78.96	1.9
3	126.7	8.12	225.5	10.39	139.37	8.12	214.22	9.87	139.37	8.93	225.5	10.91
4	153.9	6.7	185.6	10.42	169.29	6.7	194.88	10.94	138.51	7.37	194.88	10.94
5	65.7	4.74	84.5	2.36	72.27	4.74	88.73	2.48	59.13	4.27	88.73	2.48
6	76.8	4.08	103.3	4.35	84.48	4.08	108.46	4.57	69.12	4.24	108.46	4.57
7	50.2	2.53	78.8	0.16	55.22	2.53	82.74	0.17	55.22	2.78	82.74	0.17
8	44.8	2.47	59.3	1.3	49.28	2.47	62.27	1.37	40.32	2.72	62.27	1.37
9	48.1	2.32	65.7	1.49	52.91	2.32	68.99	1.56	43.29	2.55	68.99	1.56
10	89.7	4.91	163.2	6.26	98.67	4.91	155.04	5.95	98.67	5.4	163.2	6.26
11	56.9	2.24	70.7	2.8	62.59	2.24	74.23	2.94	51.21	2.46	74.24	2.94
12	112.6	5.42	142.6	2.75	123.86	5.42	149.73	2.89	101.34	4.88	149.73	2.89
13	106.9	6.28	127.8	2.7	117.59	6.28	134.19	2.84	96.21	5.65	134.19	2.84
14	54.9	3.14	62.4	1.42	60.39	3.14	65.52	1.49	60.39	3.45	65.52	1.49
15	48.8	4.43	55.2	1.38	53.68	4.43	57.96	1.45	53.68	3.99	57.96	1.45
16	59.2	3.98	95.9	0.74	65.12	3.98	100.7	0.78	65.12	4.38	100.7	0.78
17	74.5	5.32	121.6	3.06	81.95	5.32	127.68	3.21	67.05	5.85	127.68	3.21
18	94.6	3.69	107	2.98	104.06	3.69	112.35	3.13	102.17	3.32	112.35	3.13
19	47	3	65.4	0.62	51.7	3	68.67	0.65	42.3	2.7	68.67	0.65
20	54.6	3.87	71	0.01	60.06	3.87	74.55	0.01	57.72	3.48	74.55	0.01
21	90.1	3.31	81.2	5.12	99.11	3.31	85.26	5.38	99.11	2.98	85.26	5.38
22	95.2	4.25	128.3	3.89	104.72	4.25	134.72	4.08	104.72	3.83	134.72	4.08
23	80.1	3.79	135	4.73	88.11	3.79	135.39	4.97	87.47	4.17	141.75	4.97
24	68.7	2.99	98.9	1.86	75.57	2.99	103.85	1.95	75.57	2.69	103.85	1.95
25	62.3	3.1	66.7	7.41	68.53	3.1	63.37	7.04	68.53	3.41	66.7	7.41
Central	1901.5	102.97	2586.1	81.72	2091.65	102.97	2663.52	83.42	1901.5	102.96	2692.66	85.14

to increase i.e. $\gamma_r = 1$, $\beta_r = 1.10$. Furthermore, the manager expects the units output have 5% changes. Then we have, $L_{rj}^o = 0.95$, $U_{rj}^o = 1.05$.

In the second case, reallocation for two inputs is considered, i.e. $D = \{1, 2\}$. Note that to illustrate results theoretically, second input is considered discretionary. Actually, the DM goal is to reallocate the current resources just by changing inputs among branches. Hence $\alpha_1 = \alpha_2 = 1$. Also, amount of the inputs change for each branch is 10% i.e. $L_{ij}^I = 0.9$, $U_{ij}^I = 1.1$. With this reallocation, the DM does not interest to reduce output value and he (she) predicts increase the maximum (at most) 10% for total output and 5% for each unit, therefore, $L_{rj}^o = 1$, $U_{rj}^o = 1.05$, $\gamma_r = 1$, $\beta_r = 1.10$. Let $w_1 = w_2 = \dots = w_{25} = \frac{1}{25}$ in both cases.

Input-output values after resource allocation in both cases are reported in Table 3. It can be seen that inputs and outputs values changes occurs in the range

proposed by the DM. Also, inputs and outputs for efficient and inefficient units have changed to improve the organization overall CW-profit.

The resulting of the CW-profit values and the efficiency scores of branches are reported in Table 4. The values p_j and p'_j , obtained by solving model (3) in evaluating the central unit, represent the CW-profit of DMU_j before and after allocation, respectively. Also, the values θ_j and θ'_j , obtained by solving model (2), represent the efficiency scores of DMU_j before and after allocation, respectively. As it can be seen in Table 2, the efficiency scores of all units and also the central unit are increased and efficient units also remain efficient. The CW-profit of branches are

TABLE 4. Results of numerical example

DMU	Case I				Case II			
	p_j	p'_j	θ_j	θ'_j	p_j	p'_j	θ_j	θ'_j
1	-0.0237	-0.0205	0.359	0.418	-0.0246	-0.0136	0.343	0.575
2	-0.0176	-0.0154	0.431	0.494	-0.0178	-0.0098	0.423	0.62
3	0	0	1	1	-0.0015	0	1	1
4	-0.0196	-0.0109	0.849	0.923	-0.0181	-0.0006	0.849	0.992
5	-0.0213	-0.0188	0.5	0.566	-0.023	-0.0138	0.436	0.618
6	-0.0127	-0.0085	0.712	0.809	-0.0128	-0.0036	0.709	0.915
7	-0.0146	-0.0128	0.078	0.095	-0.0143	-0.0154	0.078	0.085
8	-0.0125	-0.0107	0.431	0.502	-0.0126	-0.0092	0.427	0.566
9	-0.0113	-0.0093	0.516	0.57	-0.011	-0.0068	0.516	0.659
10	0	0	1	1	0	0	1	1
11	-0.0097	-0.0069	0.805	0.86	-0.0088	-0.0027	0.805	0.902
12	-0.0321	-0.0281	0.401	0.451	-0.0315	-0.0169	0.401	0.59
13	-0.0362	-0.0328	0.367	0.418	-0.0371	-0.0237	0.346	0.502
14	-0.019	-0.0174	0.366	0.414	-0.0194	-0.0214	0.346	0.37
15	-0.0225	-0.0211	0.393	0.444	-0.0248	-0.0236	0.306	0.353
16	-0.018	-0.0156	0.243	0.3	-0.019	-0.0207	0.232	0.247
17	-0.0159	-0.0118	0.628	0.757	-0.0175	-0.011	0.583	1
18	-0.0247	-0.0214	0.562	0.617	-0.0233	-0.0196	0.562	0.739
19	-0.0164	-0.0149	0.229	0.267	-0.017	-0.011	0.215	0.332
20	-0.0243	-0.0232	0.003	0.004	-0.0255	-0.0237	0.003	0.004
21	-0.02	-0.0164	0.729	0.799	-0.0185	-0.0153	0.729	1
22	-0.0182	-0.0137	0.677	0.735	-0.0173	-0.0125	0.677	0.914
23	-0.0038	0	1	1	-0.0031	0	1	1
24	-0.0146	-0.0117	0.573	0.617	-0.0138	-0.0107	0.573	1
25	0	0	1	1	0	0	1	1
central	-0.4088	-0.3419	0.591	0.658	-0.4123	-0.2856	0.588	0.714

increased in first case but in the latter case, CW-profits of units does not necessarily increase. For example, DMU_{14} has the amount of profit -0.0194 before allocation, that it is reduced to -0.0214 after allocation. However, in the proposed method the DM's aim is to improve the overall profit of the organization, which is improved in both cases substantially. We have $P_{central} = -0.4088 < -0.3419 = P'_{central}$ in the first case, and $P_{central} = -0.4123 < -0.2856 = P'_{central}$ in the second case. However, if the DM is interested in improving the CW-profit of units, it can be fulfilled by imposing the constraints $\sum_{r=1}^s p_{rj}y_r^c - \sum_{i \in D} q_{ij}x_i^c - \sum_{i \in ND} v_{ij}x_{ij} \geq p_j$, for all j to models (9) and (11). On the other hand, Table 4 shows that, all efficient units 3, 10, 23 and 25 remain efficient and the highest profit is attained by to the same unit after allocation. It should be mentioned that our algorithm terminates in a reasonable time since there are usually a few inputs and outputs

and our method just requires to solve linear programming problems. For solving the proposed models we applied CPLEX solver.

6. Conclusion. In this paper, the problem of resource allocation and target setting was formulated as a linear programming problem. First, using the SBM non-radial model, an interpretation was presented to calculate a virtual profit for units and also for the organization. Note that, the sum of the CW-profits of unit is equal to organization overall CW-profit and inputs and outputs weights were chosen commonly for all units. One advantage of non-radial SBM model to traditional radial DEA models such as CCR and BCC is that the slacks values of inputs and outputs are also considered in calculating the efficiency scores. In other words, mixed inefficiency can be distinguished from technical efficiency in the SBM model. Moreover, positive values for input-output weights are obtained by solving this model. Such weights are independent of the choice of ϵ in CCR and BCC models.

After that, two linear models were presented. In the first model, the maximum CW-profit of the organization is calculated in the best case. In the second model, the production plans for the next period are indicated in a way that the amount of the profit is not getting worse and the profit of the units with less contribution in calculating the total profit is improved as much as possible.

The resulting allocation have the following features: The proposed method covers all types of allocation, and the DM can control the changes of resources and determine some target setting plans for each unit as well as the overall system. Hence, the DM has adequate flexibility to change the resources and target setting. Since we assume that all units are homogeneous and under the supervision of the same organization, it is acceptable to choose a common price for the production and costs of resources as well. Hence, resource are allocated to units in a way that the units are evaluated with a common set of weights in our proposed method. Also, the highest CW-profit can be obtained for the central unit after the allocation. This requirement guarantees the stability of the decision and ensures that the DMs production plans do not deteriorate the overall CW-profit. The DM is interested to allocate resources to units in a way that the CW-profit of the central unit, in the best case, is not getting worse.

Since the results of the allocation and target setting are in the range characterized by the DM, the feasibility of production plans is guaranteed for the next period. We implemented our method for solving a simple numerical example and empirical application, considering different cases including allocation of extra resources, resource reallocation and allocating less resources, and the results look acceptable and fair in practice.

As we addressed in this paper, an important DEA result is the resource allocation and target setting. DEA thus provides significant information from which analysts and managers derive insights and guidelines to enhance the existing performance of the system. Regarding this fact, effective and methodological analysis and interpretation of DEA solutions is very critical.

Emrouznejad and De Witte [7] underlined that in large and complicated data sets, a standard process could facilitate performance assessment and help to (1) translate the aim of the performance measurement to a series of small tasks, (2) select homogeneous DMUs and suggest an appropriate input/output selection, (3) detect a suitable model, (4) provide means for evaluating the effectiveness of the

results, and (5) suggest a proper solution to improve the efficiency and productivity of DMUs.

However, in large and complicated data sets, the DEA results need interpretation for the transformation of mathematical terms into managerial insights to assess and improve the performances of inefficient DMUs. The significant amount of data that come by DEA results are open to further detailed analysis for the derivation of interesting insights and guidelines. Many of the data mining and information visualization techniques are very effective tools for this analysis. Therefore, proposing a framework, which is based on the integration of DEA results with data mining and information visualization techniques can be considered in the future researches.

As an agenda for future research, we can mention three issues. First, in this study, we consider the convexity and constant return to scale axioms. However, in some applications, these axioms cannot apply. Hence, it would be good if we consider the impact of the convexity axiom and present a resource allocation model without these axioms. Second, in this study, the aim is to allocate available resources among units in a way that the overall virtual profit of the organization is maximized. But, maximizing the overall profit of a system do not guarantee that the revenue and cost efficiencies are also maximized. Therefore, proposing a method that optimizes the total revenue and cost functions in order to reach the best performance of the system can be developed. Third, in this study, we have not considered the uncertainty issue. Based on the importance of this issue in some empirical examples, extending resource allocation models with the uncertainty assumption of data can be useful.

REFERENCES

- [1] A. D. Athanassopoulos, Goal programming & data envelopment analysis (godea) for target-based multi-level planning: allocating central grants to the greek local authorities, *European Journal of Operational Research*, **87** (1995), 535–550.
- [2] J. E. Beasley, Allocating fixed costs and resources via data envelopment analysis, *European Journal of Operational Research*, **147** (2003), 198–216.
- [3] A. Charnes and W. W. Cooper, [Programming with linear fractional functionals](#), *Naval Research Logistics Quarterly*, **9** (1962), 181–186.
- [4] A. Charnes, W. W. Cooper and E. Rhodes, [Measuring the efficiency of decision making units](#), *European Journal of Operational Research*, **2** (1978), 429–444.
- [5] A. Dehnohalaji, M. Ghiyasi and P. Korhonen, Resource allocation based on cost efficiency, *Journal of the Operational Research Society*, **68** (2017), 1279–1289.
- [6] J. Du, L. Liang, Y. Chen and G. B. Bi, Dea-based production planning, *Omega*, **38** (2010), 105–112.
- [7] A. Emrouznejad and K. De Witte, Cooper-framework: A unified process for non-parametric projects, *European Journal of Operational Research*, **207** (2010), 1573–1586.
- [8] L. Fang and H. Li, Centralized resource allocation based on the cost–revenue analysis, *Computers & Industrial Engineering*, **85** (2015), 395–401.
- [9] M. J. Farrell, The measurement of productive efficiency, *Journal of the Royal Statistical Society, Series A (General)*, 253–290.
- [10] S. Gattoufi, G. R. Amin and A. Emrouznejad, A new inverse dea method for merging banks, *IMA Journal of Management Mathematics*, **25** (2014), 73–87.
- [11] B. Golany, F. Phillips and J. Rousseau, Models for improved effectiveness based on dea efficiency results, *IIE Transactions*, **25** (1993), 2–10.
- [12] B. Golany and E. Tamir, Evaluating efficiency–effectiveness–equality trade-offs: A data envelopment analysis approach, *Management Science*, **41** (1995), 1172–1184.
- [13] G. R. Jahanshahloo, J. Sadeghi and M. Khodabakhshi, [Fair ranking of the decision making units using optimistic and pessimistic weights in data envelopment analysis](#), *RAIRO-Operations Research*, **51** (2017), 253–260.

- [14] G. R. Jahanshahloo, J. Sadeghi and M. Khodabakhshi, [Proposing a method for fixed cost allocation using dea based on the efficiency invariance and common set of weights principles](#), *Mathematical Methods of Operations Research*, **85** (2007), 1–18.
- [15] P. Korhonen and M. Syrjänen, Resource allocation based on efficiency analysis, *Management Science*, **50** (2004), 1134–1144.
- [16] F. Li, Q. Zhu and L. Liang, Allocating a fixed cost based on a dea-game cross efficiency approach, *Expert Systems with Applications*, **96** (2018), 196–207.
- [17] F. Li, Q. Zhu and L. Liang, A new data envelopment analysis based approach for fixed cost allocation, *Annals of Operations Research*, 1–26.
- [18] F. H. Lotfi, A. Hatami-Marbini, P. J. Agrell, N. Aghayi and K. Gholami, Allocating fixed resources and setting targets using a common-weights dea approach, *Computers & Industrial Engineering*, **64** (2013), 631–640.
- [19] S. Lozano, G. Villa and B. Adenso-Díaz, Centralised target setting for regional recycling operations using dea, *Omega*, **32** (2004), 101–110.
- [20] S. Lozano and G. Villa, Centralized resource allocation using data envelopment analysis, *Journal of Productivity Analysis*, **22** (2004), 143–161.
- [21] N. Nasrabadi, A. Dehnokhalaji, N. A. Kiani, P. J. Korhonen and J. Wallenius, [Resource allocation for performance improvement](#), *Annals of Operations Research*, **196** (2012), 459–468.
- [22] M. Pervin, G. C. Mahata and S. K. Roy, An inventory model with declining demand market for deteriorating items under a trade credit policy, *International Journal of Management Science and Engineering Management*, **11** (2016), 243–251.
- [23] M. Pervin, S. K. Roy and G. W. Weber, [A two-echelon inventory model with stock-dependent demand and variable holding cost for deteriorating items](#), *Numerical Algebra, Control & Optimization*, **7** (2017), 21–50.
- [24] M. Pervin, S. K. Roy and G.-W. Weber, [Analysis of inventory control model with shortage under time-dependent demand and time-varying holding cost including stochastic deterioration](#), *Annals of Operations Research*, **260** (2018), 437–460.
- [25] M. Pervin, S. K. Roy and G. W. Weber, An integrated inventory model with variable holding cost under two levels of trade-credit policy, *Numerical Algebra, Control & Optimization*, **8** (2018), 169–191.
- [26] M. Pervin, S. K. Roy and G. W. Weber, [Multi-item deteriorating two-echelon inventory model with price-and stock-dependent demand: A trade-credit policy](#), *Journal of Industrial & Management Optimization*, **15** (2019), 1345–1373.
- [27] J. Sadeghi and A. Dehnokhalaji, A comprehensive method for the centralized resource allocation in dea, *Computers & Industrial Engineering*, **127** (2019), 344–352.
- [28] K. Tone, [A slacks-based measure of efficiency in data envelopment analysis](#), *European Journal of Operational Research*, **130** (2001), 498–509.
- [29] P. Wanke, C. Barros and A. Emrouznejad, [A comparison between stochastic dea and fuzzy dea approaches: revisiting efficiency in angolan banks](#), *RAIRO-Operations Research*, **52** (2018), 285–303.
- [30] Q. Wei, J. Zhang and X. Zhang, An inverse dea model for inputs/outputs estimate, *European Journal of Operational Research*, **121** (2000), 151–163.
- [31] J. Wu, Q. An, S. Ali and L. Liang, Dea based resource allocation considering environmental factors, *Mathematical and Computer Modelling*, **58** (2013), 1128–1137.
- [32] L. Xiaoya and C. Jinchuan, [A comprehensive dea approach for the resource allocation problem based on scale economies classification](#), *Journal of Systems Science and Complexity*, **21** (2008), 540–557.
- [33] H. Yan, Q. Wei and G. Hao, [Dea models for resource reallocation and production input/output estimation](#), *European Journal of Operational Research*, **136** (2002), 19–31.
- [34] M. Zahedi-Seresht, G.-R. Jahanshahloo, J. Jablonsky and S. Asghariniya, [A new monte carlo based procedure for complete ranking efficient units in dea models](#), *Numerical Algebra, Control & Optimization*, **7** (2017), 403–416.

Received April 2018; 1st revision July 2018; Final revision July 2019.

E-mail address: std.j.sadeghi@khu.ac.ir

E-mail address: mogshu@gmail.com

E-mail address: Akram.Dehnokhalaji@khu.ac.ir